RESEARCH DEPARTMENT

Low-noise amplifiers for u.h.f. reception

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LOW-NOISE AMPLIFIERS FOR U.H.F. RECEPTION

Research Report No. G-090

(1964/23)

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Section Title Page

SUMMARY 1

1. INTRODUCTION 1

2. TUNNEL DIODE AMPLIFIER 4

2.1. Principle of Operation 4
2.2. Alternative Amplifier Configurations 5
2.3. Experimental Work 7

2.3.1. 200 Mc/s Hybrid Amplifier 7
2.3.2. 600 Mc/s Hybrid Amplifier 8

2.4. Advantages and Disadvantages of Tunnel-Diode Amplifiers 9

3. THE QUADRUPOLE AMPLIFIER 9

3.1. Historical Introduction 9
3.2. Principle of Operation 10
3.3. Power Requirements 12
3.4. Noise Figure 12
3.5. Bandwidth 13

4. THE VARIABLE-CAPACITANCE-DIODE PARAMETRIC AMPLIFIER 13

4.1. Principle of Operation 13
4.2. Alternative Circuit Configurations 16
4.3. Gain, Bandwidth and Noise 17
4.4. Practical Requirements and Performance 18

5. DISCUSSION 19

6. CONCLUSIONS 21

7. REFERENCES 21

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SUMMARY

Noise in broadcast receivers generally increases as the frequency increases, whereas noise from external sources decreases. In receivers designed for u.h.f. using conventional valve or transistor circuits, the noise is about 10 dB greater than the aerial noise. There is therefore an obvious economic advantage to be gained by reducing receiver noise at u.h.f., particularly for special purposes such as rebroadcast reception. Several new low-noise amplifying techniques which offer a solution to this problem have recently been developed for use in other fields such as radar and radio-astronomy, and the application of these techniques to television reception at u.h.f. is discussed. Three essentially different types of low-noise amplifier - the tunnel diode, the quadrupole and the varactor-diode parametric amplifier - are described in some detail. Of these, the parametric amplifier appears to offer the best compromise between good performance and complexity at the present state of development.

1. INTRODUCTION

In receiving a transmitted ultra-high frequency (u.h.f.) signal, the signal/noise ratio will be degraded by noise contributions not only from the input circuits of the receiver but also from the receiving aerial. When compared with the noise level in u.h.f. valve amplifiers and crystal mixers, this aerial noise is of little significance.[1] But in recent years new systems for low-level u.h.f. amplification have become available which, with varying degrees of performance and complexity, afford means of reducing the noise contribution of a receiver to a level equal to or even below that of the aerial noise. Before describing some of these low-noise systems it is useful to discuss the ultimate economic limit to the receiver performance which is set by aerial noise at u.h.f.[2,3]

We shall use the concept of 'equivalent noise temperature'; when applied to any impedance with a resistive term, R, this refers to the value of T which gives the observed mean square noise voltage $\overline{v_n^2}$ according to the expression

$$\overline{v_n^2} = 4\ kTRB$$

where B is the bandwidth of the measuring device and k is Boltzmann's constant $1{\cdot}38\times10^{-23}$ watt-seconds per degree. If R is the resistance of a physical conductor, then T is also its physical absolute temperature. If, alternatively, R is the radiation resistance of a lossless aerial, then T is not the temperature of the aerial itself, but represents the effective mean temperature of the objects surrounding the aerial, integrated over its radiation pattern at the received frequency.

At high frequencies the effective temperature of an aerial may be many thousands of degrees, because of radiation from man-made sources, atmospheric disturbances and extra-terrestrial objects. At u.h.f., however, the radiation from these sources is so small that the principal source is the ground at 290°K. The mean sky temperature is about 300°K at 300 Mc/s but falls as $f^{-2{\cdot}4}$, where f is the frequency. Although the sky contains several intense, discrete radio-sources at much higher temperatures, such as large stars and colliding nebulae, the resulting radiation (excepting that of the Sun) will not greatly affect the temperature of an aerial unless its gain is very high and a celestial source falls within the narrow aerial beam. For typical communications or broadcast receiving aerials, the sources will be lost in the general background radiation of our galaxy.[2,3] At 600 Mc/s, the effective sky temperature is about 150°K near the Milky Way, falling to about 10°K near the galactic poles, when observed with an aerial having a single lobe about 20° wide (18 dB gain) directed nearly vertically upwards. Near the horizon, the sky temperature increases owing to losses in the atmosphere, while the effective ground temperature falls below 290°K owing to partial reflexion of the sky - particularly in open country. As a result, a typical u.h.f. broadcast receiving aerial directed at the horizon will have a mean effective temperature of 150 - 220°K; this will be increased by up to 60°K if the sun passes within the acceptance beam of the receiving aerial. There may, of course, be a contribution from man-made interference, such as car ignition interference. This has not yet been fully investigated at u.h.f. but is known to be only comparable with present-day receiver noise when a low-gain aerial is used near traffic - a situation unlikely in a fringe area or a re-broadcast installation.

If there are losses in the aerial feeder system, with a physical temperature of T_o, such that the power transmission factor is α, then the modified noise temperature of the aerial and feeder is

$$\begin{aligned} T'_a &= \alpha\, T_a + (1 - \alpha)\, T_o \\ &= T_a + (1 - \alpha)(T_o - T_a) \end{aligned}$$

For example, if the feeder is at 290°K, the equivalent temperature of the aerial is 190°K and the loss is 3 dB ($\alpha = 0{\cdot}5$), the noise temperature at the end of the feeder becomes 240°K. In addition, the wanted signal power will be reduced by factor α.

The performance of a receiver is often described by the noise factor, which is the ratio of the total noise power (source + receiver) to the noise power from the source alone when the source resistance is at an effective temperature of 290°K. We may therefore express the noise factor, N, as

$$N = \frac{T_r + 290}{290}$$

[illegible]

NdB = 10 log10 N [illegible]

$$\text{or} \qquad T_r = 290(N - 1)\ {}^\circ\mathrm{K}$$

where T_r represents the equivalent temperature of the receiver - sometimes called the excess noise temperature or effective input temperature.[4] T_r is in many ways a better description of the receiver noise than the noise factor, since its definition does not involve a standard source temperature. Thus, the total noise in a receiving system is described by attributing to the source resistance an effective temperature which is the sum of the aerial temperature, T_a, the contribution from feeder loss, $(1-\alpha)(T_o-T_a)$, and the excess noise temperature of the receiver, T_r.

Improvements in broadcast receiver design which lead to a receiver temperature appreciably lower than the aerial temperature are of diminishing value. A reasonable compromise may be reached by making the equivalent temperature of the receiver equal to that of the aerial. At 100 Mc/s this temperature is about 1000°K corresponding to a noise factor of 6·5 dB, a figure which can be achieved by conventional valve circuits; but at 600 Mc/s the temperature is about 240°K (corresponding to a noise factor of 2·6 dB). This performance cannot be reached in conventional u.h.f. valve amplifiers, transistor amplifiers or crystal mixers; an equivalent receiver temperature of 2000°K (9 dB noise factor) is possible with careful design, while for current commercial u.h.f. television receivers in Europe the figure is often about 5500°K (13 dB noise factor) because economic and other factors affect the design, e.g. a mass-produced valve is widely used as a first r.f. amplifier. There is therefore considerable room for improvement in u.h.f. receivers. Transistors are already available which give a slightly better performance than valves and, as the limitations of transistors still appear to be those of manufacturing techniques, this trend may continue. Transistors and valves, however, amplify by the same process of controlling a current flow by means of an injected charge, and it seems unlikely that a major break-through in their noise performance will be achieved. It is the entirely different methods of amplification that have recently been developed which can offer a tenfold reduction in receiver noise temperature.[5]

Some of these new devices are, like the valve and the transistor, basically non-reciprocal, such as the travelling-wave tube and the quadrupole amplifier; these operate by enhancing periodic variations which are impressed at one end of a beam of electrons so that an amplified signal may be extracted at the other end. There are, on the other hand, essentially reciprocal regenerative devices such as the tunnel diode, the parametric amplifier and the maser which derive power gain from a negative resistance. In these latter amplifiers, the input and output powers flow at the same pair of terminals and can be separated only by directional couplers, circulators or isolators which distinguish the signals by their direction of flow. Negative resistance is inherent in the tunnel diode but, in the diode parametric amplifier, it arises through variation of inductance or capacitance in one or more resonant circuits. In the maser the action is similar to that of the parametric amplifier, with the resonant circuits replaced by the quantum-mechanical energy levels of selected materials.

Since all of these amplifiers have effective temperatures of the same order as room temperature, a further useful reduction of noise would seem to be possible by reducing their physical temperature. A considerable improvement can be obtained by cooling the parametric amplifier and the maser since most of their noise is of thermal origin in inevitable losses. The tunnel diode, however, is limited in its performance by the shot noise from the d.c. bias and is hardly improved by cooling. The thermal noise in electron beam devices can be substantially reduced by special measures in construction such as by coupling a cooled resistive load to the beam before the signal is applied. A constant, relatively cool load (about 30°K at u.h.f.) can be obtained, without refrigeration, by directing a high-gain aerial towards the Pole Star.

2. TUNNEL DIODE AMPLIFIER

2.1. Principle of Operation

When suitably biased, a tunnel diode (or Esaki diode) exhibits a negative resistance which can be used to neutralize resistive losses in a signal-frequency circuit, giving an effective gain in signal power. In this negative-resistance condition, the basic mechanism controlling the flow of current is very rapid in operation and involves no sources of noise other than the 'shot' noise associated with the current itself. Low-noise amplification at u.h.f. is therefore possible with this device.

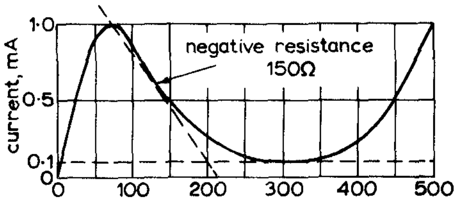


Fig. 1 - Typical tunnel-diode characteristic

The static characteristic of a typical tunnel diode is shown in Fig. 1; the general shape of the curve is a function of the material used for the junction but the magnitude of the current at any voltage, and hence the magnitude of the negative resistance, is determined by the area of the junction. Thus, the peak current may be, say, between 1 mA and 50 mA and the magnitude of the negative resistance between 200 ohms and 2 ohms.

Restricting attention to the region between the peak current and the 'valley' current, where the slope of the characteristic is negative, an equivalent circuit can be given which is valid for small signals; this is shown in Fig. 2. $R-$ is the intrinsic negative resistance, say -150 ohms for a working bias of 100 mV, C_d the diode junction capacitance, typically 5 pF, R_s the ohmic resistance of the connexions to the junction, typically 1 ohm, and L_s the lead inductance, typically 5 nH. Consideration of the behaviour of this circuit when the terminals are short-circuited leads to an inequality

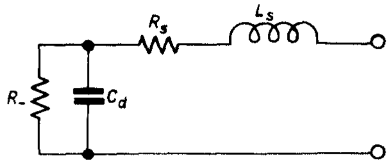


Fig. 2 - Equivalent circuit of tunnel diode

$$|R-| > R_s > L_s/C_d\,|R-|$$

which must be satisfied if both exponential and oscillatory instability are to be avoided.[6] Fig. 3 shows diagrammatically the regions of stability and instability when the device is short-circuited. With a practical termination, such as an amplifier circuit, stability can only be investigated rigorously with a complete knowledge of the broad-band equivalent circuit of the termination. The stability conditions may then be examined in relation to the effective values of R_s and L_s when modified to include the terminal impedance. Nevertheless, having first ensured stability at zero frequency and at the operating frequency, it may be sufficient in practice to examine the impedance conditions near the cut-off frequencies of the signal and bias networks.

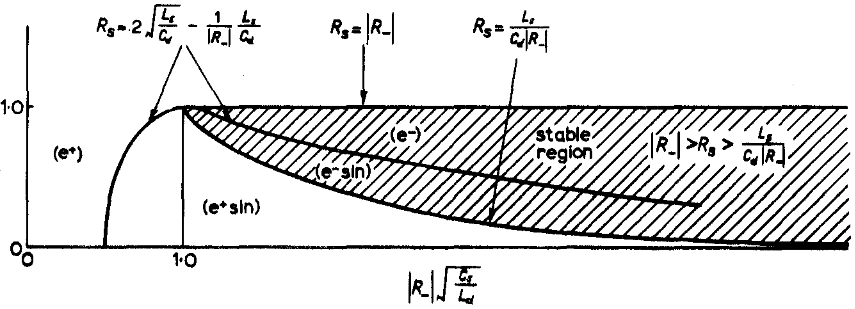
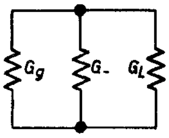


Fig. 3 - Stability regions for tunnel diode on short circuit

2.2. Alternative Amplifier Configurations

A single tunnel diode may be used as an amplifier simply by connecting it in parallel with a source and a load, as shown in Fig. 4. This arrangement, however, has a number of disadvantages, as follows. If the gain is to be greater than 6 dB, the conductances presented to the source and the load cannot both be positive. In this case, if, for example, the output admittance is matched to that of the load, the amplifier will not be unconditionally stable at the input terminals as the input conductance will be negative. Thus, in order to obtain high gain with stability, the source admittance must be maintained within close limits. Departure from the correct source admittance will not only affect the gain but also the noise factor as the output match will quickly deteriorate. Under controlled laboratory conditions, a gain of 36 dB and a noise figure of 5 dB have been obtained over a 6 Mc/s band at 530 Mc/s.[7]

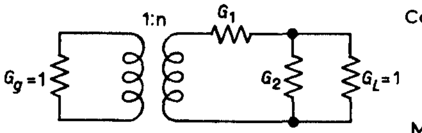
Fig. 4
Simple tunnel-diode amplifier



power gain $= \frac{4G_g G_L}{(G_g+G_L-|G_-|)^2}$
$\longrightarrow \infty$ as $G_- \longrightarrow -(G_g+G_L)$
input conductance $= G_L-|G_-|$
output conductance $= G_g-|G_-|$

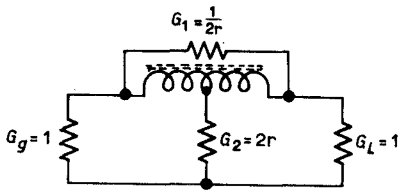
One way of reducing the difficulties is to use two negative resistance elements. It is then possible to obtain gain between a nominally matched source and load together with unconditional stability at either pair of terminals.[8] This relieves some of the restrictions on the source admittance, but only if the load admittance is closely controlled. A simple circuit with this property is shown in Fig. 5 and another which uses a hybrid transformer is shown in Fig. 6. Both of

Fig. 5
Simple two-diode circuit



Conditions for power match
$G_1 = \frac{1}{G_2} - G_2$
$n_2 = (1-G_2{}^2)^{-1}$
Matched power gain $= \left|\frac{1-G_2}{1+G_2}\right|$
$\longrightarrow \infty$ as $G_2 \longrightarrow -1$

Fig. 6
Hybrid transformer circuit



power gain $= (\frac{1-r}{1+r})^2$
$\longrightarrow \infty$ as $r \longrightarrow -1$

these are impracticable for direct use with tunnel diodes because the circuits require negative resistances of two opposite types; one must be of the short-circuit stable (tunnel-diode) type with a characteristic as shown in Fig. 1 and the other must have the dual characteristic with voltage and current axes interchanged, thus displaying open-circuit stability. While pnpn diodes offer the latter type of characteristic, their residual reactances do not allow stability under any condition. The dual of a tunnel diode can be simulated over a limited bandwidth, however, by interposing a quarter-wave line between the diode and the amplifier.

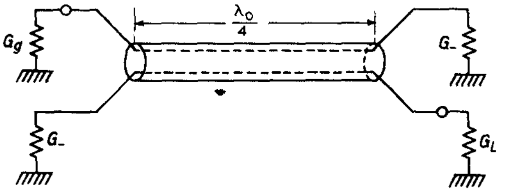


Fig. 7 - 3 dB directional coupler circuit

A more practicable arrangement employs two identical diodes and a suitable type of 3 dB directional coupler[9] as shown in Fig. 7. Here, the action may be regarded as if the tunnel diodes provide reflexion coefficients of greater than unity. By virtue of the quarter-wave line, the enhanced signals reflected by the diodes cancel at the sending end and add at the load. This arrangement is unconditionally stable provided that the load be closely matched at all frequencies. The bandwidth is limited by the coupler structure. For example, with the screened two-wire line illustrated in Fig. 7 the amplifier will have a gain of zero at twice the design frequency.

The theoretical gain-bandwidth product is most closely approached by distributing a large number of tunnel diodes along a transmission line.[10] The characteristic impedance of such a line will be positive, so that it can be correctly terminated by a real matched load but, as the propagation constant has a negative real part, it affords power gain. For gain, stability and noise factor, the 'transmission-line amplifier' is very similar to the two-diode hybrid-coupled type but the gain is considerably less dependent on the characteristics of the individual diodes.

All the amplifiers discussed so far are essentially 'reciprocal' networks and have the same gain in either direction. Reflexions from the load may therefore cause a violent change in performance. Many of the difficulties can be reduced by using non-reciprocal devices such as isolators or circulators. An isolator offers a high attenuation to any power reflected by the load, whereas a circulator diverts the reflected power so that it bypasses the negative resistance and is not re-amplified. The behaviour of a perfect three-port circulator is shown in Fig. 8.

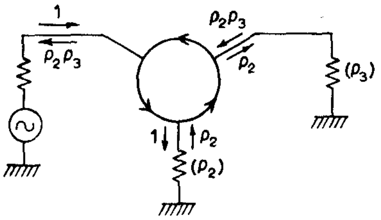


Fig. 8 - Three-port circulator: power flow from a matched source with finite voltage reflexion coefficients at the other two ports

These devices have inherent bandwidth limitations and are not perfect in their non-reciprocal action but they enable a gain of about 20 dB to be obtained with a single negative resistance device such as a tunnel diode, allowing for practicable variations of source and load admittance. A circulator was not available during the experimental work on tunnel diodes, and attention was therefore confined to a type of amplifier using two diodes.

2.3. Experimental Work

In view of the considerations in the previous section the diode circuit configuration chosen was the hybrid-coupled amplifier,[9] employing a 3 dB directional coupler as a '90° hybrid' and two tunnel diodes. Two hybrid amplifiers were constructed, one at v.h.f. (200 Mc/s), the other at u.h.f. (600 Mc/s). The purpose of the v.h.f. amplifier was to provide design and setting-up experience before embarking on the u.h.f. model.

Both of these units used the same design of coupler structure but different diode arrangements. In the v.h.f. amplifier, the tunnel diodes were shunt-tuned with coils and mounted in brass blocks which connected to the hybrid by short cables. In the u.h.f. amplifier the tunnel diodes were shunt-tuned by short inductive lines and mounted within the hybrid unit.

The coupler structure was designed to give an input/output ratio of 3 dB at an impedance, (Z_o), of 71 ohms; this value of impedance was convenient not only from a measurement standpoint but also because, using direct connexion of the currently available diodes, it provided a design gain of about 10 dB. This gain was thought to be adequate for the present investigation, since the amplifier matching requirements to maintain stability become less severe as the gain is reduced. In principle there is no reason why the diode tuned impedance should not be transformed down to match the impedance, Z_o, of the coupler in order to achieve more gain. (It should be noted that stability considerations preclude making this impedance fall below Z_o).

An experimental coupler structure was made and its push-push and push-pull impedances[11] were checked by low-frequency capacitance measurements before producing the final designs for the hybrids. In both units, the discontinuities at the hybrid ports were adjusted for maximum directivity; with the 200 Mc/s version it was also possible to check that the output ratio was unity and that the coupler characteristic impedance were symmetrical about the nominal value of Z_o.

2.3.1. 200 Mc/s Hybrid Amplifier

The admittance characteristics of Semiconductors Ltd. diodes T1925 and T1975 were measured in the tuned and untuned condition with frequency and bias as parameters; for this purpose the diodes were mounted in their final cable-connected jigs, set at $\lambda/4$ from the measuring plane of the General Radio admittance meter.

The T1975 diodes were chosen for the final amplifier as less selection was required to obtain two with the desired admittance; these diodes have the following pertinent parameters:

Resistive cut-off frequency	1200 Mc/s
Total diode capacitance	11 pF
Maximum negative conductance	8·35 mmho

These values indicate a theoretical low-frequency gain of 11 dB and a diode Q of about 2 at 200 Mc/s. Under optimum bias conditions the measured amplifier performance was as follows:

Centre frequency f_o	213 Mc/s
Half-power bandwidth	150 Mc/s (± 75 Mc/s)
Gain at f_o	8 dB
Noise factor at f_o	6·5 dB

The noise factor is that of the amplifier alone, allowance having been made for the receiver connected to the output. This was a v.h.f. field strength recording receiver (Band III) r.f. unit. The receiver input impedance was adjusted to 71 ohms at 213 Mc/s. A 5 dB attenuator was inserted between amplifier and receiver, the combination yielding a receiver noise factor of 17 dB.

2.3.2. 600 Mc/s Hybrid Amplifier

In this amplifier, General Electric diodes 1N 2939 were utilized with nominal parameters:

Resistive cut-off frequency	2200 Mc/s
Total diode capacitance	5 pF
Series Inductance	6 nH
Maximum negative conductance	6·6 mmho

These figures give a theoretical low-frequency gain of 9 dB.

Two diodes were selected which, in the shunt tuned condition, yielded a hybrid match under as nearly equal bias settings as possible. The following performance measurements were made:

(1) Matched insertion gain/frequency curves, taken with mean diode bias as parameter; these were compared with the theoretical gain/frequency curve deduced from the full diode equivalent circuit;

(2) Variation of matched insertion gain at f_o with diode bias potentials was explored;

(3) The noise factor of the amplifier, determined under varying bias conditions.

With optimum bias, the best performance was as follows:

Centre frequency f_o	600 Mc/s
Half-power bandwidth	23 Mc/s
Gain at f_o	7 dB
Noise factor at f_o	8·5 dB

For noise factor measurements the post-hybrid receiver used was a commercial u.h.f. tuner followed by a communications receiver tuned to the i.f. output frequency of the tuner (36 Mc/s). The input circuit of the tuner was modified to provide a match at f_o such that the product of the input reflexion coefficient magnitude and the hybrid voltage gain was well below unity over a bandwidth ± 30 Mc/s. The noise factor of this arrangement was 15·5 dB.

It was interesting in this connexion to note that the amplifier had appreciable gain at the frequency of the local oscillator in the receiver, so that the oscillator voltage reaching the aerial input was amplified to about 750 μV.

Further experiments were conducted, with filters in the diode circuit, to improve stability by constraining the diode resistance to the positive domain in the region between the required pass-band and the pass-band of the hybrid structure. Unfortunately, the reactance of the filters themselves gave rise to instability and the analysis of the circuit behaviour became unwieldy.

2.4. Advantages and Disadvantages of Tunnel-Diode Amplifiers

The most attractive feature of the tunnel-diode amplifier is that, with a very simple power supply, and when operating between a resistive source and load, it can provide a voltage-gain bandwidth product of several kMc/s.

The main disadvantage is that with practical terminations the wide bandwidth of the active element leads to severe stability difficulties. The stability can be improved by using non-reciprocal devices such as circulators or isolators, by using two diodes in a hybrid line structure, by using several diodes in a distributed line structure or by using a combination of these methods. But none of these methods is perfect; each imposes limitations on the performance and increases complexity.

As a low-noise amplifier, an ideal tunnel diode gives a theoretical noise factor of about 2 dB (180°K effective temperature) but, because of losses, matching errors, non-zero valley current and the need to operate near the peak current for reasons of dynamic stability, a noise factor of 5 dB is the best that can be expected in practice.

3. THE QUADRUPOLE AMPLIFIER

3.1. Historical Introduction

In 1949 Cuccia[12] described an electron-beam, high-power u.h.f. modulator which had a number of advantages including low insertion loss (about 2 dB), perfect isolation between input and output, and no frequency-pulling effect. Nine years later in 1958 Adler[13] pointed out that the valve was also a low-noise device and could form the basis of a low-noise amplifier. All that was required was a region between the input coupler and the output coupler where an anisotropic electric field varying at twice the signal frequency could produce parametric amplification. In a further letter[14] four months later he and Hrbek reported very promising results using a quadrupole structure and in October 1959 a full paper[15] was published giving the design and performance of what is known as the quadrupole amplifier, electron beam amplifier or Adler tube. All successful subsequent designs (see, for example, reference 16) have been fairly close copies of the valves described in 1959, and it seems likely that the device is reaching the limit of its performance in its present form.

3.2. Principle of Operation

Fig. 9 shows the principal elements of the quadrupole amplifier. The entire electrode structure is immersed in a uniform axial magnetic field. In the absence of a signal input, the electron beam is ideally monochromatic and exactly parallel with the magnetic field. When a signal is applied, the transverse deflexion of the electrons passing between the coupler plates causes them to gyrate about the axial field at the so-called cyclotron frequency. By adjusting the strength of the magnetic field, the cyclotron frequency may be made equal to the signal frequency. In this case, the electron orbits absorb energy and increase in radius so that the whole beam describes a conical surface, leaving the coupler with a radius proportional to the signal level. This gyrating beam then passes through the electric field set up by the pump oscillator between the four plates of the quadrupole. Amplification occurs through some of the electron orbits absorbing energy from the quadrupole field in proportion to their initial radius which therefore increases exponentially. Leaving the quadrupole, this amplified beam passes on to induce potentials in the output coupler, reducing in orbital radius as energy is given up to the load resistance. Gains of up to 30 dB can be obtained without difficulty and the gain is always zero in the reverse direction, giving unconditional stability. If the pump oscillator fails, the beam is not affected by the quadrupole and the overall forward loss of the valve may be as low as 0•5 dB.

Since the beam can both absorb and give up energy as it passes through a coupler it can be simulated by a resistive component and generator in the equivalent circuit of each coupler. By careful design of the reactive circuits feeding a coupler this resistive component may be accurately matched to the source (or load) resistance. In this condition, power can flow into or out of the coupler without reflexion, as its equivalent circuit is perfectly reciprocal and may be regarded as a perfectly-matched lossless transmission line. The non-reciprocal behaviour of the valve as a whole arises from the fact that the power entering or leaving the beam can only travel in one direction - that of the electrons forming the beam. Thus noise already present on the beam, in the form of a net random transverse motion of the electrons leaving the gun, will be completely absorbed by the input coupler and dissipated in the source resistance, while the signal power from the source

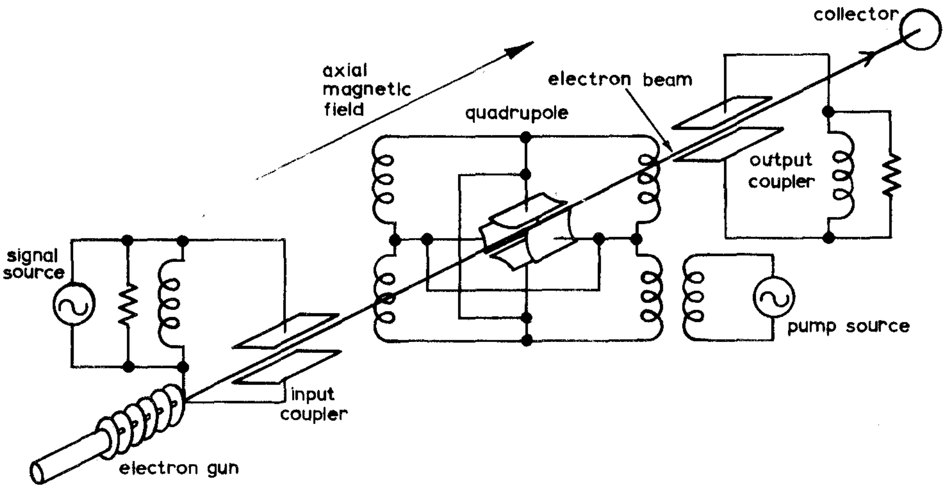


Fig. 9 - Schematic diagram of the quadrupole amplifier

will be completely absorbed by the 'cleaned' beam as it passes on towards the quadrupole. In practice, however, there is some loss in the reactive circuit feeding the input coupler, which generally takes the form of a coil connected across the coupler, forming a resonant circuit with the capacitance between the plates. Unless the shunt conductance corresponding to the coil loss is combined with a deliberately-added series resistance, so forming a matching attenuator, the source and the beam cannot be simultaneously matched. Since the removal of noise is usually more important than a slight loss of signal power, the input coupler is usually matched to the beam while the output coupler is matched to the load.

Fig. 10 shows a cross-section of the quadrupole with the polarities of the electrodes indicated for one instant of time. It can be seen that the field between the plates is zero on the axis and contains a tangential component which increases with distance from the axis. This field distribution can be represented by superposing two field patterns rotating in opposite directions at half the pump frequency. If therefore the pump frequency is twice the cyclotron frequency, then the field will be synchronous with an electron at X which will be accelerated and travel in an exponentially increasing orbit as it passes through the quadrupole, whereas an electron at Y will be decelerated and decrease in orbital radius. However, the effect on all electrons, averaged over all phases of the pump field, is a net gain, the total energy extracted from the pump field exceeding the total energy given up to it. The maximum radius of the electron beam entering the output coupler is therefore greater than that leaving the input coupler, and amplification has been achieved.

If the signal frequency ω_s is not exactly equal to one half of the pump frequency $\omega_0/2$, but differs from it by $\Delta\omega$, the electron beam will drift in phase with respect to the quadrupole field, giving successive conditions of gain and attenuation alternating at a frequency $2\Delta\omega$. The resulting fluctuations in the signal-frequency output may equally well be attributed to the addition of a modulation

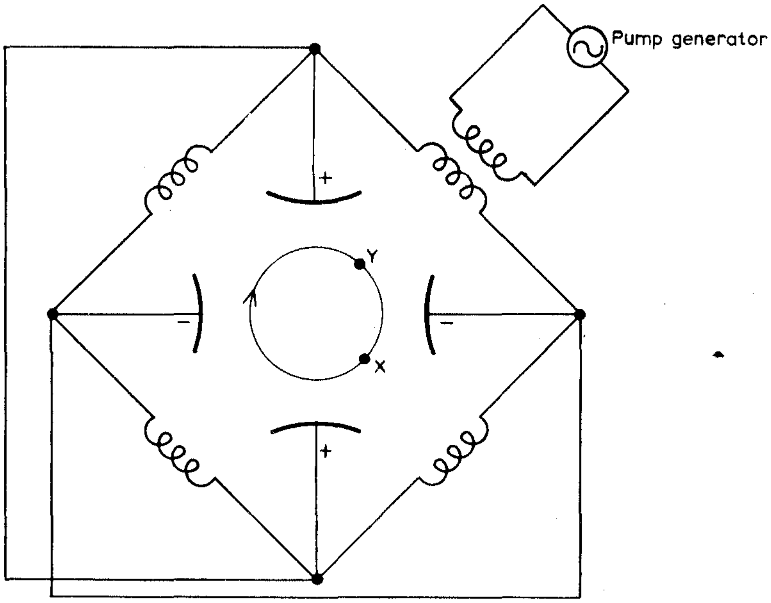


Fig. 10 - Cross-sectional view of the quadrupole

product at a frequency $\omega_o - \omega_s$, the so-called idler frequency associated with the action of all parametric devices. This effect constitutes one of the major disadvantages of the quadrupole valve when required to amplify a coherent signal, leading to difficulties in fully achieving the inherent low-noise performance, as explained later in Section 3.4.

3.3. Power Requirements

The power supplies to the quadrupole amplifier are rather more complicated than with most other valves and may be grouped under three headings:

(1) Valve electrode supplies including the cathode-heater current

(2) The electromagnet current

(3) The quadrupole pump power (at frequency $\omega_o \simeq 2\omega_s$)

Direct current supplies are required for the cathode, six anodes, two collectors, quadrupole and collector; all must be fully stabilized but, as the current taken by each is small and varies little, a conventional stabilized supply is adequate. In most designs this supply must be floating to avoid the need for isolating the couplers from earth, hence neither supply terminal is at earth potential.

Because the cyclotron frequency is directly proportional to the magnetic field it is essential to maintain the current in the electromagnet at a constant value (usually about 5A). It has been suggested[17] that a stability of ± 1% is required but practical experience with a particular valve showed that this is not good enough for broadcast applications and a figure of ± 0·5% is desirable.

The construction of the electrode supplies and the field-current supply can be achieved by standard techniques but the cost of this is quite high and forms a major part of the cost of the device. The supplies are also bulky (due to the need for transformers and chokes) and it seems unlikely that an outdoor installation in an exposed position, e.g. on a mast, would be satisfactory.

In the laboratory experiments, a conventional u.h.f. oscillator provided the pump power (60 mW at 1200 Mc/s) but the gain of the amplifier varied rapidly with change in pump power and pump frequency. If the quadrupole valve gain must be maintained within ± 4 dB of the nominal value the pump power must remain constant within ± 0·5 dB and the pump frequency must remain within ± 1 part in 10^4. For long-term unattended operation these requirements are very stringent and are likely to make the cost of the pump supply excessive.

3.4. Noise Figure

The quadrupole amplifier is inherently a low-noise valve and noise figures of better than 2 dB can be maintained in production models; indeed better than 1 dB can be achieved in principle, and it has been found possible to approach this in practice. A more important limitation of the device is the noise introduced at the idler frequency. It was explained in Section 3.2. that if a signal at ω_s is applied, one of the modulation products is an idler frequency $\omega_o - \omega_s$. It follows that, conversely, there is nothing to prevent an input signal or noise component at $\omega_o - \omega_s$ appearing in the output at ω_s, a situation analogous to the image frequency arising in a superheterodyne with no r.f. selectivity. Thus, if a coherent signal is applied, noise contributions near the image frequency as well as the signal frequency appear in the output at the signal frequency and the noise figure is 3 dB worse than the noise figure measured using a wideband noise source. This disadvantage could be largely overcome by using filters to present one aerial (pointed at the source of the wanted signal) to the valve at the signal frequency and to present another aerial (pointed at a 'cool' part of the sky) to the valve at the idler frequency. However, the improvement is unlikely to justify the consequent complication and cost of such a procedure.

The noise figure is also dependent on the input s.w.r. for, if the input coupler is not matched to the source, some of the noise on the electron beam, which may have an effective temperature of 1000°K, will be reflected back on to the electron beam and therefore appear amplified at the output. If the noise figure is not to be degraded by more than 0·2 dB, the input s.w.r. must be better than 1·1. It is difficult to maintain this condition over the full gain-bandwidth of the valve (for example 60 Mc/s for a 600 Mc/s valve) because of imperfections in the reactance compensation of the balun feeding the input coupler. In practice, the low noise figure can be maintained over about half the full bandwidth (30 Mc/s for a 600 Mc/s valve).

One further consideration which will be very important in practice is that a large number of controls must be adjusted before the best noise figure is obtained. A continuously reading noise factor meter appears to be essential and it might indeed be necessary to install one of these at each relay station in rebroadcast application. This would add further to the cost particularly if a special instrument had to be developed for the purpose.

3.5. Bandwidth

Assuming that the pump frequency is exactly double the signal frequency (that is to say, the valve is operated in the degenerate mode) the gain-bandwidth is limited by the couplers to about 20 Mc/s at 200 Mc/s and about 60 Mc/s at 600 Mc/s (measured at the points where the gain falls by 3 dB). The maximum bandwidth in the non-degenerate mode, where the mid-band frequency is half the pump frequency, is half this value. Further, because a guard band must be provided so that the inverted idler signal may be removed, the useful signal bandwidth is about one third of the total bandwidth, e.g. 20 Mc/s at 600 Mc/s.

Experiments were carried out to examine the intermodulation products as the vision frequency of a 625-line television signal approaches half the pump frequency; no interference was observed till the signal and idler bands overlapped.

4. THE VARIABLE-CAPACITANCE-DIODE PARAMETRIC AMPLIFIER

4.1. Principle of Operation

As its name implies, a parametric amplifier provides power gain at the frequency of an applied signal by actively varying one or more parameters of an otherwise passive circuit. The way in which this occurs can be illustrated by considering a parallel resonant circuit consisting of inductance L, capacitance C and resistance R, in which the current executes a damped oscillation. The total energy stored in the circuit is

$$E = \tfrac{1}{2} Q^2/C + \tfrac{1}{2} \Phi^2/L$$

where Q, the instantaneous charge on the capacitor, and Φ, the instantaneous flux in the inductor, will execute damped oscillations in quadrature as energy is exchanged between them and gradually dissipated in the resistance R. This expression shows that the total energy in the circuit can be increased by reducing the value of C or L or of both. Capacitance may be reduced by pulling the capacitor plates apart and inductance may be reduced by pulling the coil turns apart; in either case work is expended against the attractive force of the electric or magnetic field and energy is transferred to the circuit. The reduction of L or C does not need to be continuous; it can be performed by pumping the component in a cyclic manner, reducing its value when the field is at a maximum and increasing it to its original value when the field passes through zero so that energy is not returned. There will thus be two cycles of pumping for every cycle of the circuit oscillation. If the power supplied by the pump equals that dissipated in the resistor R, all damping is removed, and the circuit will oscillate continuously at its resonant frequency. The effect of pumping at twice the signal frequency may therefore be described as equivalent to that of a negative resistance at the signal frequency. By controlling the magnitude of this negative resistance and connecting to a source and a load, this arrangement can provide power gain in a very similar manner to that described in Section 2 for the tunnel diode and, although the requirements are less stringent, similar precautions are necessary to avoid instability.[18] A convenient variable-capacitance circuit element which can be varied extremely rapidly is provided by the 'varactor' diode - a semiconductor diode in which the reactance of the reverse-biased barrier capacitance is roughly proportional to the square root of the applied voltage. Mathematical analysis of the behaviour of this type of circuit has been made[19,20,21] and copious references to experimental work are available.[19]

The simple example given above illustrates the basic mechanism of parametric amplification, but the use of a single tuned circuit is of limited practical application in low-noise amplifiers. In order to show some of the more important advantages to be gained by the application of the principle, the more general arrangement of a parametric converter, as shown in Fig. 11, will be described.

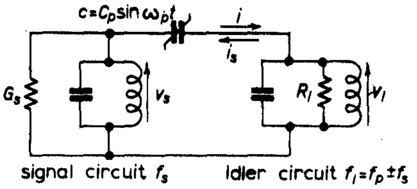


Fig. 11
Circuit of parametric amplifier

Consider a parallel tuned circuit energized at its resonant frequency $f_s = \omega_s/2\pi$ and connected to a capacitor which is being varied in capacitance - or 'pumped' at a frequency $f_p = \omega_p/2\pi$;

$$v_s = V_s \cos(\omega_s t + \phi_s)$$

$$c = C_p \sin \omega_p t$$

The charge on the capacitor at each instant will be equal to the product of the capacitance and the voltage, and the current flowing into the capacitor will be the time-derivative of this charge:

$$q = cv$$

$$i = dq/dt$$

Owing to the multiplicative process, the current will have components i_{p+s}, i_{p-s} at the sum and difference frequencies and, owing to the differentiation, each component will have an amplitude proportional to its own angular frequency;

$$i_{p+s} = \tfrac{1}{2} C_p V_s (\omega_p \pm \omega_s) \cos (\overline{\omega_p \pm \omega_s}\, t \pm \phi_s)$$

Either of these two current components may be made to develop a voltage and dissipate power by adding, in series with the variable capacitor, a parallel circuit resonant at the chosen frequency with a shunt load resistor. This circuit, and the chosen frequency, are referred to as 'idle' in a parametric amplifier, by analogy with the idler gear in a gear-chain, which transmits essential power though nothing is driven from its shaft. In this case, the voltage and power in the idler load resistance R_I are respectively

$$v_I = R_I i_{p\pm s}$$

and $$P_I = \tfrac{1}{2} R_I i^2_{p\pm s} = \tfrac{1}{8} C_p^2 V_s^2 R_I (\omega_p \pm \omega_s)$$

But this idler voltage will also appear across the variable capacitor and will interact with the pump frequency in the same way as the signal voltage, producing a current flowing towards the generator containing a difference-frequency component i_s at the signal frequency;

$$i_s = \mp \tfrac{1}{4} C_p^2 V_s R_I \omega_s (\omega_p \pm \omega_s) \cos (\omega_s t + \phi_s)$$

the two values corresponding to the idler being at the sum or difference frequency respectively. We see that, depending on this choice, i_s may be either in phase or in antiphase with the signal voltage, indicating two possible directions of power flow and two possible signs for the input conductance of the circuit; these are

$$G_{\text{in}} = \pm \tfrac{1}{4} C_p^2 R_I \omega_s (\omega_p \pm \omega_s)$$

$$P_s = \pm \tfrac{1}{8} C_p^2 V_s^2 R_I \omega_s (\omega_p \pm \omega_s)$$

Comparing the power flowing away from the variable capacitor at the signal and idler frequencies, and remembering that their sum must equal the pump power input P_p since there are, by assumption, no other losses and energy is conserved overall, we have

$$\frac{P_s}{\omega_s} = \frac{\mp P_I}{\omega_p \pm \omega_s} = \frac{\pm P_p}{\omega_p}$$

More general power relations have been established by Manley and Rowe[19,22], who have treated parametric amplifier systems with idler circuits for all conceivable intermodulation products of the signal and pump frequencies.

4.2. Alternative Circuit Configurations

For the three frequencies discussed in the previous section, the expressions show that the power flowing at each frequency is proportional to the frequency itself. The way in which the choice of pump and idler frequencies affects the general properties of the circuit can be illustrated by the following examples, the direction of power flow being indicated in Fig. 12.

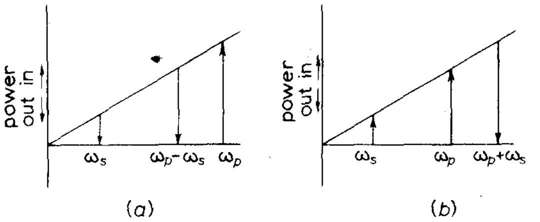


Fig. 12
Energy relationships

(a) Amplifier and inverting up-converter
(b) Non-inverting up-converter

(i) Non-degenerate parametric amplifier; Fig. 12(a)

The pump power is applied at the highest of the three frequencies, usually many times the signal frequency, and the idler frequency is the difference between them, $\omega_p - \omega_s$. Power therefore flows from the diode at the signal and idler frequencies and a negative component of conductance appears across both circuits. Either of these lower frequencies may be used for regenerative amplification or oscillation and the circuit is completely reciprocal, unless an isolator or circulator is used. Other properties are discussed in the next section where an amplifier of this type is described.

(ii) Frequency-inverting up-converter or down-converter; Fig. 12(a)

If, in arrangement (i), the signal is applied at the lowest frequency but the output is extracted at the idler frequency, we have an up-converter. The output spectrum is a transposed mirror-image of the input spectrum. A down-converter is obtained by interchanging the roles of the signal and idler frequencies.

(iii) Degenerate parametric amplifier

As (ii) but with the pump frequency equal to twice the signal frequency so that the idler frequency becomes identical with the signal frequency and no separate idler circuit is required. As before, the idler spectrum is the mirror-image of the signal spectrum. Hence in this case the signal and idler spectra overlap and their components will interfere with each other. This is embarrassing when dealing with coherent signals in, for example, television, but is of little consequence in radar or radio-astronomy, where this arrangement gives an advantage in noise performance. The action is similar to that of the quadrupole amplifier discussed in Section 3.

(iv) Non-inverting up-converter or down-converter; Fig. 12(b)

If the idler frequency is the sum of the pump and signal frequencies, $\omega_p + \omega_s$ we have a non-inverting up-converter. As is clear from the power relationship, power cannot appear at the idler frequency unless power is applied at the signal frequency; the terminal conductances are therefore both positive and regeneration does not occur. Nevertheless, there is stable power gain between the signal and idler frequencies with a corresponding loss in the reverse direction (down-converter). The non-reciprocal behaviour of this type of converter appears attractive but this is offset by the need to provide a low-noise second-stage amplifier at a frequency several times that of the signal.

(v) Hybrid-coupled parametric amplifier[23]

As (i) but using two varactor diodes coupled by a 90° hybrid line, an arrangement similar to that described in Section 2 for the tunnel diode. The circuit has a great advantage over the tunnel diode circuit in that it can be made non-reciprocal by making the varactor diodes share a common idler-frequency load and by pumping them in quadrature. The idler currents are then additive for signals flowing towards the load but cancel one another for signals reflected by the load, so reducing the reverse gain to less than unity. The performance of this arrangement is said to be the same as that of an amplifier of type (i) using a three-port circulator.

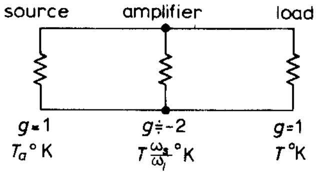
There are several other arrangements which make use of the basic mechanism of parametric energy conversion[19,24,25] but, for the following sections, consideration will be confined to the non-degenerate parametric amplifier described in (i) above.

4.3. Gain, Bandwidth and Noise

The gain of a varactor-diode parametric amplifier is the same as that of the tunnel-diode amplifier (given in Section 2) but, in this case, the magnitude of the negative conductance depends on the power supplied to the diode by the pump. It will be clear from Section 4.1. that, since the negative admittance at the signal-frequency input is proportional to the positive impedance at the idler frequency, the negative conductance has the same absolute bandwidth as the idler resistance. When equal source and load conductances are added, the bandwidth as an amplifier will decrease as the gain is increased and the voltage-gain-bandwidth product will therefore be equal to the bandwidth of the idler circuit, f_I/Q_I.

In an ideal parametric amplifier there is only one source of noise - that of the idler resistance. The power relations show that the noise power produced by this resistance at the signal frequency is reduced in the ratio of the signal frequency ω to the idler frequency ω_I. The negative input conductance, therefore, is an effective temperature $T\omega_s/\omega_I$ where T is the idler circuit temperature. In a simple reciprocal amplifier of high gain, as in Fig. 13, the magnitude of the negative

Fig. 13
Simple parametric amplifier



conductance is about twice that of the source and load, and the load temperature has to be considered as part of the total receiver temperature. Assuming the load and the amplifier to be at the physical temperature T throughout, the equivalent amplifier temperature is

$$T_{\mathrm{eff}} = T\left(1 + \frac{2\omega_s}{\omega_I}\right)$$

If a circulator is used, as shown in Fig. 14, the load noise is isolated from the amplifier (provided that the source is well matched) and the negative conductance is nearly equal to the source conductance. In these circumstances the effective amplifier temperature is simply that of the negative conductance,

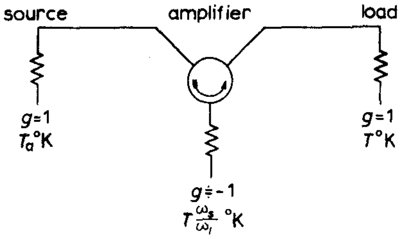


Fig. 14
Parametric amplifier with circulator

$$T_{\mathrm{eff(circulator)}} = T\frac{\omega_s}{\omega_I}$$

The expressions given above will be somewhat inaccurate in practice since the amplifier and load temperatures may not be exactly equal, the circulator imperfections will introduce signal loss and noise, and there will be a contribution from the thermal noise in the 'spreading resistance' losses of the parametric diode. These are discussed more fully in the references.[20,21]

4.4. Practical Requirements and Performance

If the gain of the amplifier is to be, say, 20 dB, it will vary by 1 dB if the pump output varies by 0·1 dB, so that the pump oscillator is required to have a high order of stability. To achieve a high gain-bandwidth and low noise-temperature, the pump frequency should be made as high as is possible consistent with the limit set by the diode losses. Furthermore, the idler circuit must be tuned to resonance with as little extra reactance as possible in order to keep the value of Q_I low and maintain a wide bandwidth.

For example, in a 600 Mc/s amplifier using a microwave 'pill' diode, pumped at 8 kMc/s, the idler circuit consisted simply of the residual reactance of the diode and its mount. Using a circulator (but without cooling), the excess noise temperature was about 170°K (2 dB noise figure) and the bandwidth was 15 Mc/s with a mid-band gain of 20 dB. The amplifier was followed by a grounded-grid triode with a noise temperature of 2600°K (10 dB noise figure). The pump power required was a few milliwatts provided by a klystron oscillator. By increasing the pump power the gain could be increased but instability occurred at a gain of about 35 dB, due mainly to imperfections in the circulator; these, however, were negligible at a gain of 20 dB. The bandwidth over which the circulator performance was maintained was about twice that of the amplifier at 20 dB gain so that, at all frequencies outside the pass-band determined by the idler circuit, the negative conductance was low enough to keep the loop gain round the circulator well below unity and preserve stability.

In the event of the pump oscillator failing during operation, the amplifier gain will simply drop to slightly less than unity. This is because the diode will then present an open circuit at the signal frequency and the signal power will be passed on to the load by the circulator after nearly complete reflexion at the diode port. Fig. 15 shows a series of photographs of part of a television picture transmitted on Channel 44 with an attenuator in the aerial feeder to reduce the input signal to about 100 μV. The effective source temperature was 290°K in these conditions, so that the signal/noise ratio is inversely proportional to the noise figure as defined in Section 1. Fig. 15(a) shows the unpumped condition with an effective noise figure of 10 dB due to the succeeding amplifier. In Fig. 15(b) the parametric amplifier is pumped to 20 dB gain, giving a noise figure of 2 dB. Fig. 15(c) shows the effect of subsequently adding 8 dB extra aerial attenuation, which effectively reduces the signal/noise ratio to that of the unpumped condition shown in (a). It should perhaps be pointed out that the poor definition of this picture is not connected with the performance of the parametric amplifier.

In order to assess the susceptibility of the amplifier to adjacent-channel interference, a square-wave modulated signal was adjusted in amplitude and frequency to give perceptible interference on the Channel 44 test card transmission, the wanted signal level being 500 μV for this test. When the interfering signal was set to a frequency other than 10 Mc/s of the tuned frequency, this caused perceptible interference when the interfering/wanted signal ratio was 0 dB or less but, at 20 Mc/s or more from the tuned frequency, the ratio became greater than 10 dB. In the unpumped condition, the corresponding ratio was slightly lower since, in this condition the signal-frequency bandwidth was greater.

Although the klystron pump oscillator used for these tests had adequate stability for the purpose, it may be possible in the future to use a low-frequency pump oscillator followed by varactor-diode stages in a passive high-efficiency multiplier; this would reduce the bulk and may increase the long-term reliability of the parametric amplifier as a whole.

5. DISCUSSION

It is clear that the factor limiting the sensitivity of u.h.f. broadcast receivers, using conventional valve techniques, is the receiver noise. A reduction of about 10 dB would be advantageous before external noise levels predominate. Of the many new devices which offer such improvements, we have described three in some detail. Of these, the tunnel diode amplifier appears to be the simplest, but it suffers from the disadvantage of it being difficult to ensure adequate gain and stability in practical circumstances. This difficulty can be obviated by using passive non-reciprocal devices such as circulators but, at u.h.f., these are costly and cumbersome by comparison with the simple tunnel-diode amplifier. If a circulator is to be used, it would seem to be more appropriate to follow it by a varactor-diode

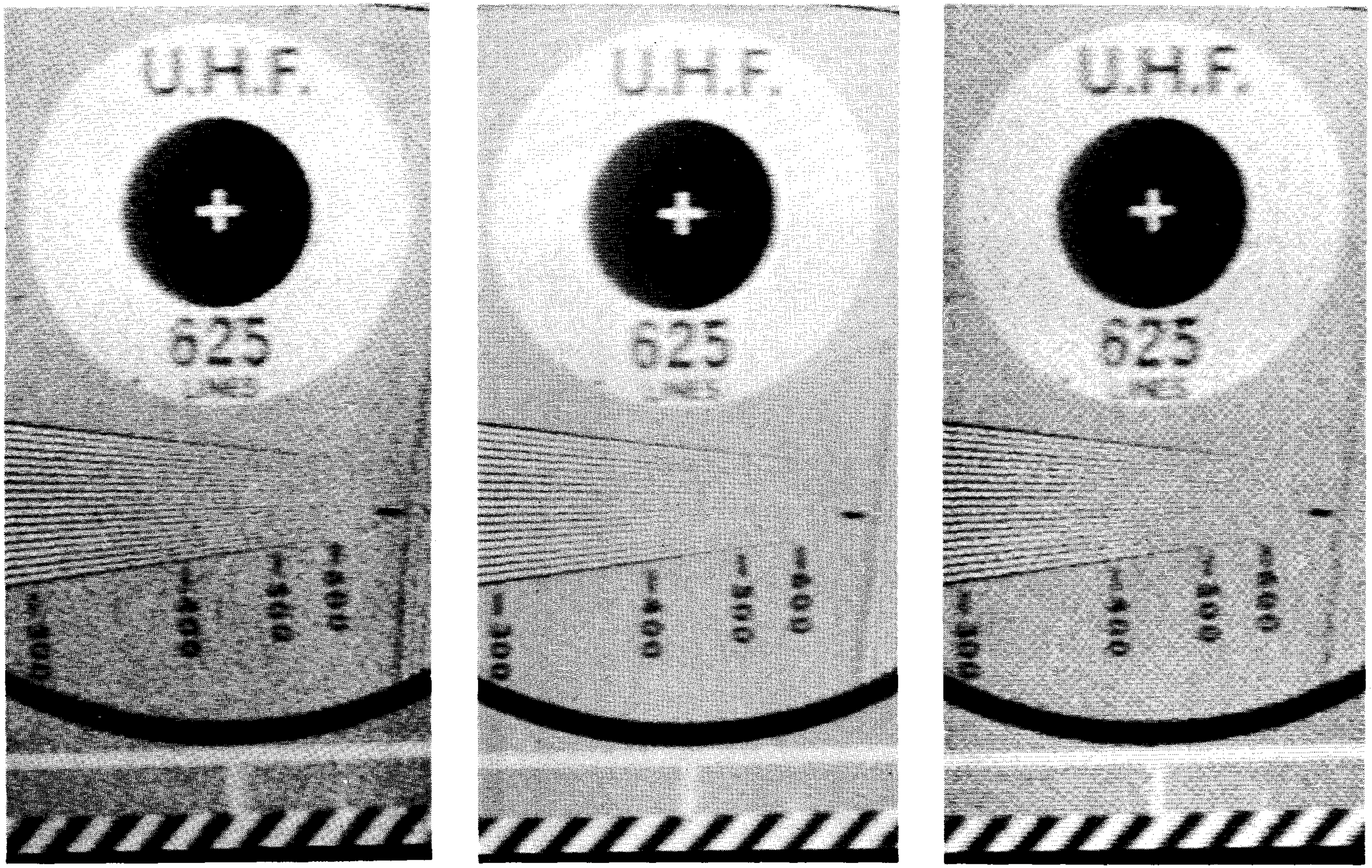


Fig. 15 - Pictures received from Channel 44 transmission using parametric amplifier

PUMP OFF

(a) Gain	0 dB
(a) N.F.	10 dB
Signal Atten.	0 dB
Tot. Noise Temp.	2900°K
S/N Ratio	24 dB

PUMP ON

(b) Gain	20 dB
(b) N.F.	2 dB
Signal Atten.	0 dB
Tot. Noise Temp.	460°K
S/N Ratio	32 dB

PUMP ON

(c) Gain	20 dB
(c) N.F.	2 dB
Signal Atten.	8 dB
Tot. Noise Temp.	460°K
S/N Ratio	24 dB

parametric amplifier which gives a much better performance. The only additional complexity is that a pump oscillator of good stability must be provided; the parametric amplifier itself is as simple as a tunnel-diode amplifier. An alternative to the complexity of the parametric amplifier and circulator is offered by the quadrupole electron beam amplifier, which is basically non-reciprocal, but here the complexity re-appears in the form of the highly stable power supply requirements, and in avoiding interference from the idler channel.

6. CONCLUSIONS

On balance, the performance and stability of the diode parametric amplifier, used with a circulator, make it the most attractive at the present time for special purposes where the signal level is particularly low. These conditions may arise at some receiving sites used for u.h.f. television rebroadcasting. It is possible, however, that future improvements in circuit techniques and in the development of cheaper circulators may transfer attention to the tunnel diode as a general-purpose low-noise amplifier.

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